A Brief History of Oscillators and Hair Styles of European Men

AASU Math/CS Colloquium
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Images and quotes:

www-groups.dcs.st-and.ac.uk
(U. St. Andrews, Scotland)

Galileo Galilei
1564 – 1642

What Galileo learnt about the pendulum:

Pendulums return (almost) to their release heights.

All pendulums eventually come to rest; lighter ones get there faster.

Period is independent of the bob's weight.

Period is independent of the amplitude.

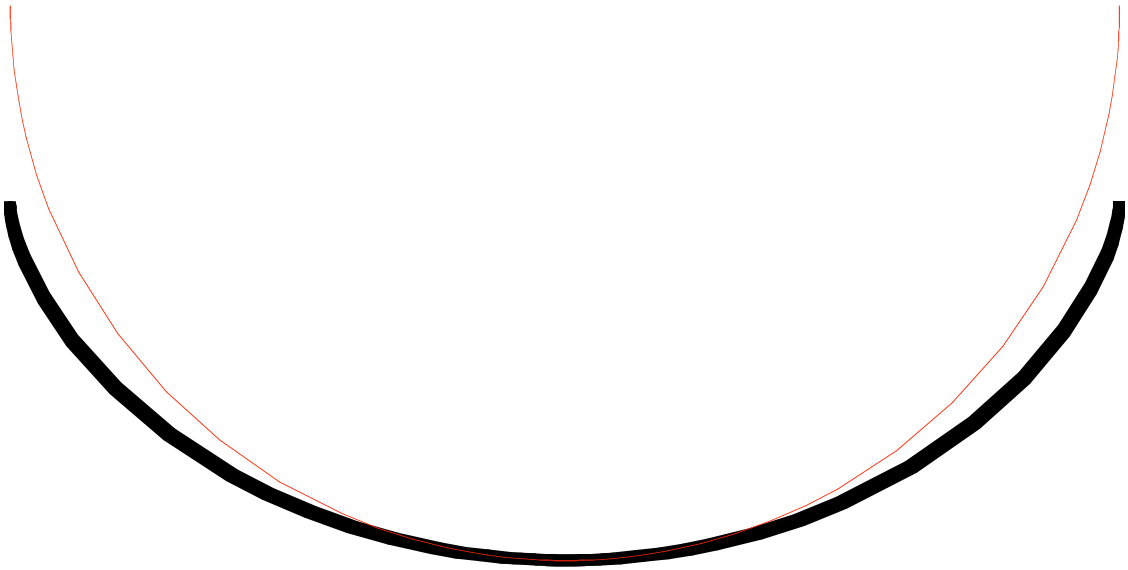
The square of the period varies directly with the length.

Christiaan Huygens
1629 – 1695

Patented the first pendulum clock in 1656.

Published *Horologium Oscillatorium* in 1673.

Huygens discovered the *tautochrone* property of the cycloid.



John Bernoulli later discovered (in 1696) that it has
the *brachistochrone* property too.

The pendulum equation:

$$L\,\theta'' = -g \sin\theta$$

When θ is small,

$$L\,\theta'' \approx -g\theta$$

(Approximately) simple harmonic motion
with period $2\pi\sqrt{L/g}$.

The exact period is given by an *elliptic integral*:

$$\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{4\,d\theta}{\sqrt{1 - \sin^2(\frac{1}{2}\,\theta_0)\sin^2\theta}}$$

(Not known until much later.)

Robert Hooke
1635 – 1703

Formulated Hooke's law of elasticity in 1660.

"No portrait of Hooke is known to exist. He has been described as
a lean, bent and ugly man,
and so he may not have been willing to sit for a painting of his portrait."

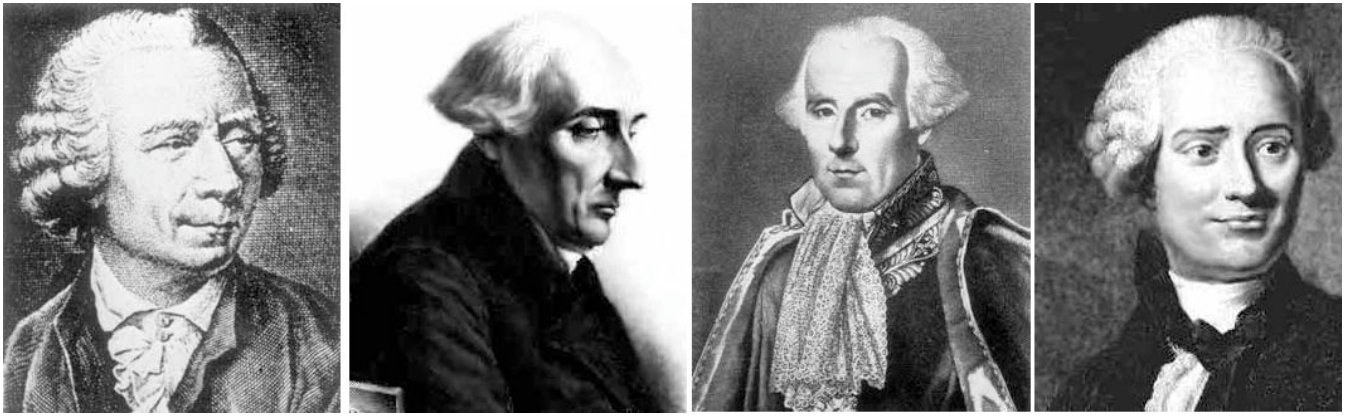
The spring-mass equation:

$$m\,x'' = -k\,x$$

Simple harmonic motion with period $2\pi\sqrt{k/m}$.

Fast Forward …

1700s: Euler, Lagrange, Laplace, d'Alembert, …
Acoustics, heat conduction, optics, hydrodynamics,
calculus of variations...



Early 1800s: Carnot, Fourier, Cauchy, ...



The steam engine and the birth of thermodynamics;
major progress in analysis.

Hermann von Helmholtz
1821 – 1894

The Helmholtz Oscillator

Lord Kelvin
(William Thomson)
1824 – 1907

KELVINATOR

..the Finest in Electric Refrigeration

If you knew as much as we do about the 20th Anniversary Kelvinator, we are sure you would agree with us, frankly, Without Boasting—and with no intention of disparaging the other fine electric refrigerators being built today, we do believe the new 1934 DeLuxe Kelvinator is the finest in electric refrigeration.

Of course, we are proud of it! Why shouldn't we be? We know how it is engineered—how it is built—of what fine materials it is made—and how it will perform. We know the tradition behind it. We know the brilliant record our engineers and designers have made as pioneers in the industry during the past twenty years—the largest experience in the industry. And finally, we know this is their greatest achievement.

The Kelvinator management extends a cordial invitation to the readers of Fortune in all the corners of the globe—an invitation to visit the nearest dealer and inspect the 20th Anniversary Kelvinators. Wherever Fortune goes, a Kelvinator dealer is very likely to be found. . . . KELVINATOR CORPORATION, 14081 Plymouth Road, Detroit, Michigan. Factories also in London, Ontario, and London, England. 1914 1934

Lord Rayleigh
(John William Strutt)
1842 – 1919

Discovered argon in 1895.
For this he won a **Nobel Prize** in 1904.

Rayleigh modeled the oscillations of a clarinet reed.

$$y'' = ay' - b(y')^3 - cy$$

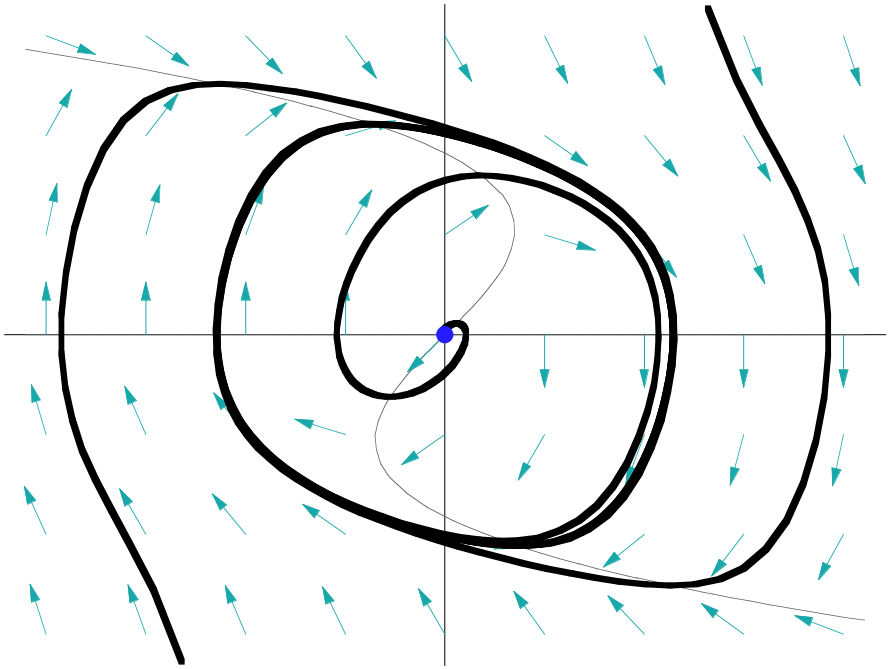
Jules Henri Poincaré
1854 – 1912

A Theorem of Poincaré:

Suppose that $\langle f(x, y), g(x, y)\rangle$ is a polynomial vector field and that $\Gamma = \{\langle x(t), y(t)\rangle : t \geq 0\}$ is a curve satisfying the differential equation

$$\frac{d}{dt}\langle x(t),\ y(t)\rangle = \langle f(x(t),\ y(t)),\ g(x(t),\ y(t))\rangle \quad \text{for } t \geq 0.$$

If Γ is contained in a closed, bounded set in which $\langle f(x, y), g(x, y)\rangle$ is never zero, then the set of limit points of Γ is a simple closed curve that satisfies the same differential equation.

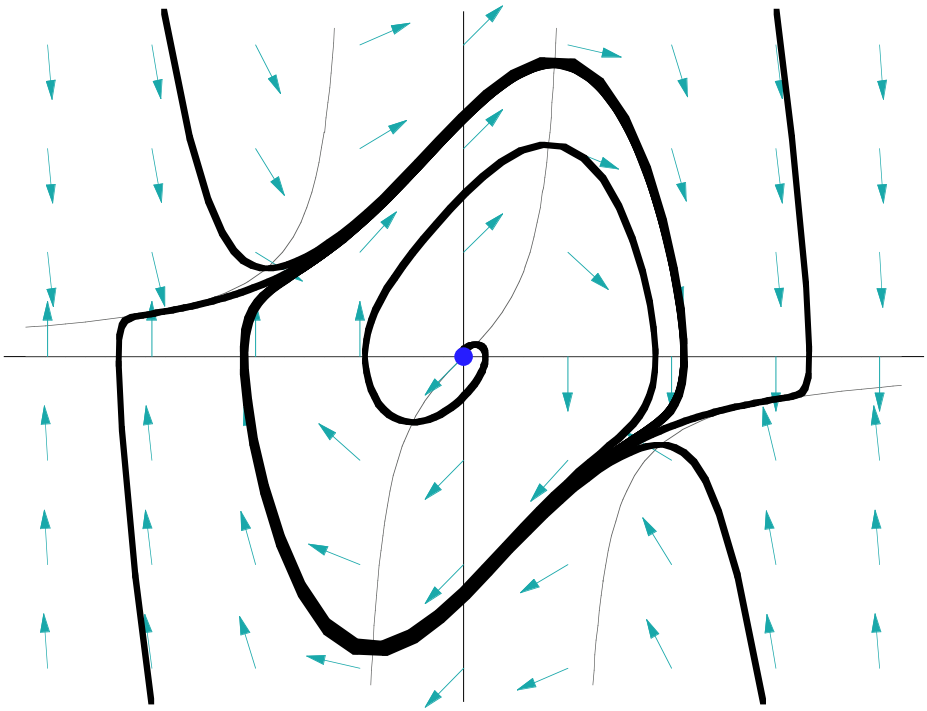


Meanwhile … Faraday, Maxwell, et al. are working on electricity and magnetism. Later in the century, the first vacuum tubes are built, which would eventually lead to radio, television, and computers.

It was good that scientists were working to understand electricity, because …

Balthasar van der Pol
1889 – 1959

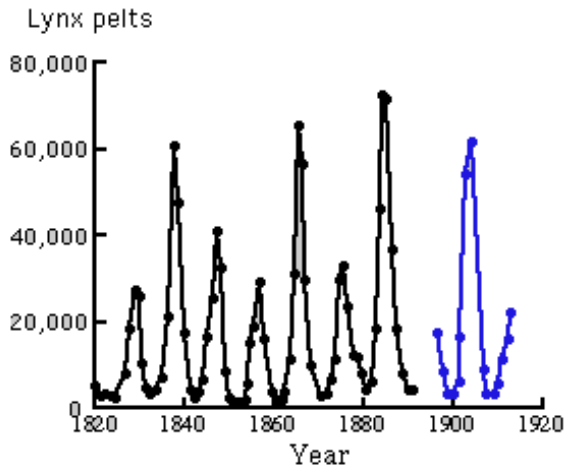
"Van der Pol built a number of electronic circuit models of the human heart to study the range of stability of heart dynamics. His investigations with adding an external driving signal were analogous to the situation in which a real heart is driven by a pacemaker."

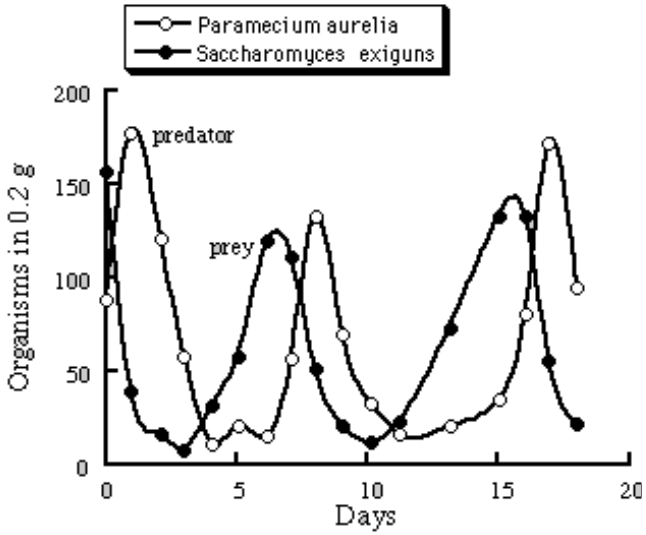


Alfred Lotka
1880 – 1949

Vito Volterra
1860 – 1940

Lotka is best known for the predator-prey model he proposed—at the same time as but independent from Volterra. In 1924, he published *Elements of Mathematical Biology*. He later left science to work for an insurance company.

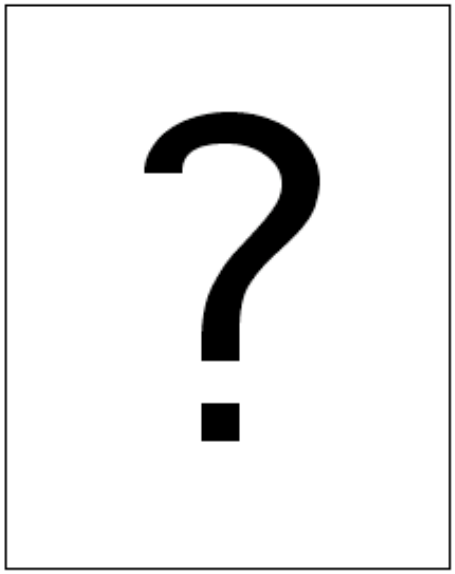




Prey: $\dfrac{dp}{dt}\Big/ p \;= a_1 - b_1 p - c_1 q,$

Predator: $\dfrac{dq}{dt}\Big/ q \;= -a_2 + b_2 p$

Eberhard Hopf
1902 – 1983



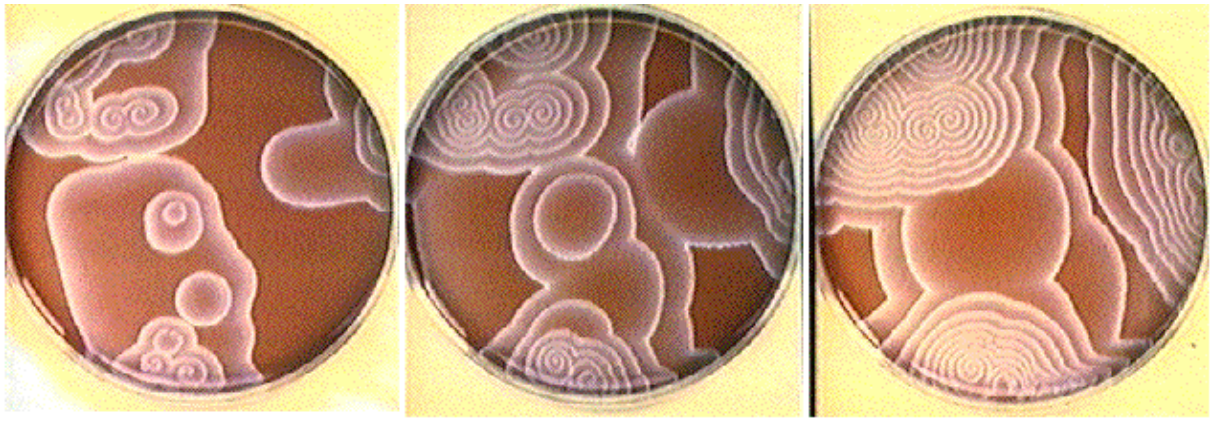
A. Andronov
?? – 1952

Bifurcation Theory
(ca. 1940)

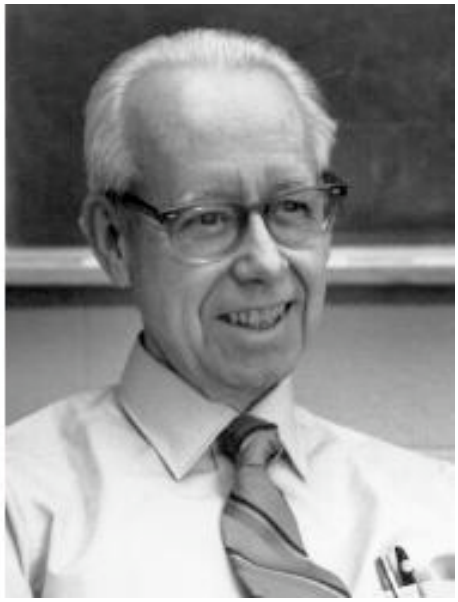
Boris P. Belousov
1893 – 1970

Anatol M. Zhabotinski
1938 –

"The modern history of the study of oscillating chemical reactions in the liquid phase began in Russia in 1951, when B. P. Belousov discovered temporal oscillations in the ratio [Ce(IV)]/[Ce(III)]during the cerium-ion-catalyzed oxidation of citric acid in acidic bromate."



In 1980 the **Lenin Prize** *was awarded to Belousov, Zhabotinsky, V. I. Krinsky, G. R. Ivanitsky and A. Zaikin for their work on the BZ reaction. Belousov had died in 1970.*



Richard Field Richard Noyes

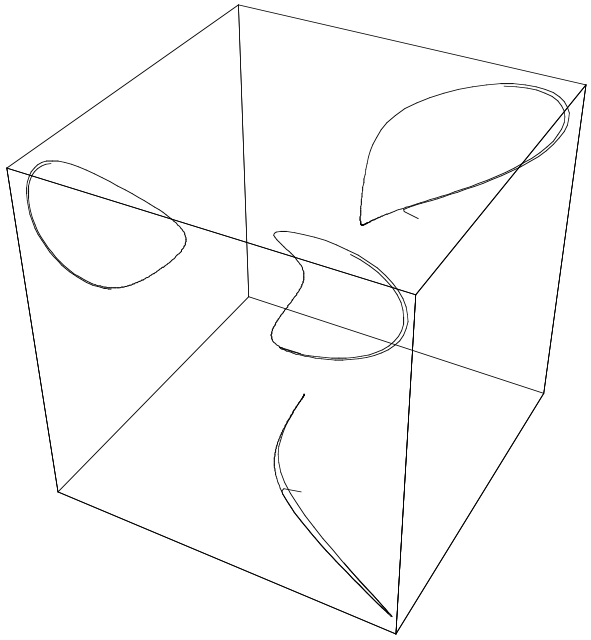
University of Oregon (ca. 1970)

Developed a mathematically tractable model of the BZ reaction, which became known as …

The Oregonator

$$A + Y \to X + P$$
$$X + Y \to 2\,P$$
$$A + X \to 2\,X + 2\,Z$$
$$2\,X \to A + P$$
$$2\,Z \to Y$$

$$x' = \varepsilon\,(q\,y - x\,y + x\,(1 - x))$$
$$y' = \mu\,(-q\,y - x\,y + z)$$
$$z' = x - y$$



Ilya Prigogine
1917 –

1977 Nobel Prize in Chemistry

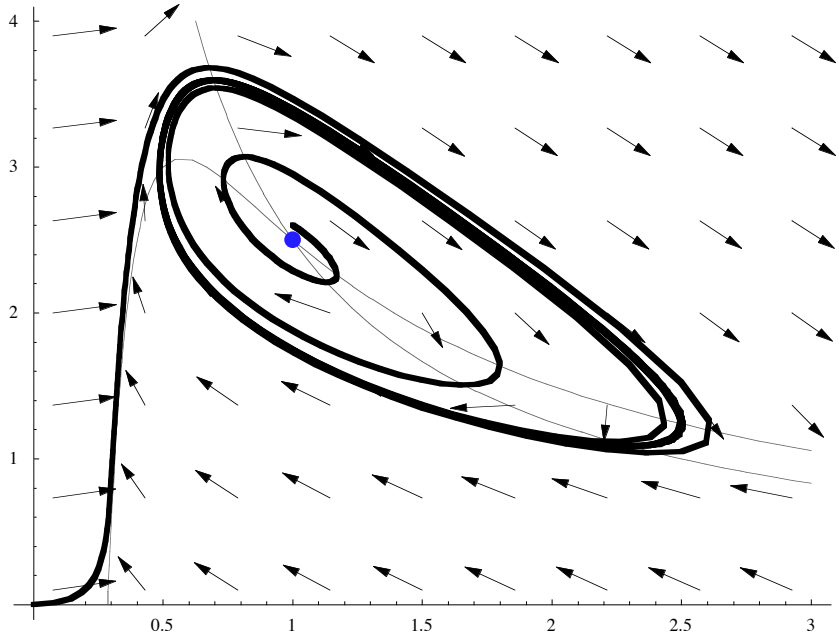
"I was born in Moscow, on the 25th of January, 1917 — a few months before the revolution. My family had a difficult relationship with the new regime, and so we left Russia as early as 1921. For some years (until 1929), we lived as migrants in Germany, before we stayed for good in Belgium. It was at Brussels that I attended secondary school and university. I acquired Belgian nationality in 1949."

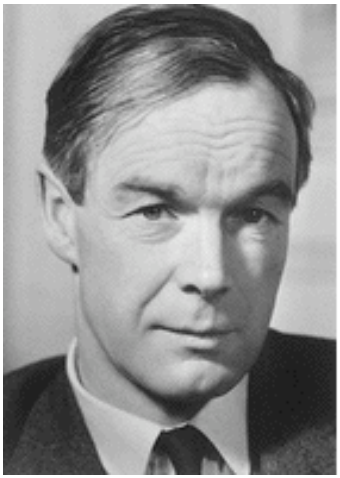
The Brusselator

$$A \to B, \quad B + D \to C, \quad 2\,B + C \to 3\,B, \quad B \to P$$

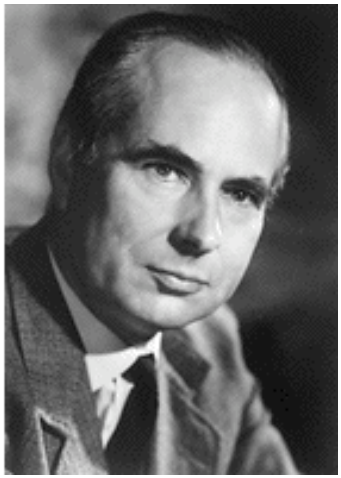
$$x' = x^2\,y - (\mu + 1)\,x + \gamma$$

$$y' = -x^2\,y + \mu\,x$$





Sir Alan Lloyd Hodgkin
1914 – 1998



Sir Andrew Fielding Huxley
1917 –

1963 Nobel Prize in Medicine

Explained electrochemical impulses that travel along the giant squid axon.

His grandfather, Thomas Hodgkin, and uncle, Robin Hodgkin, were historians and to begin with Alan hesitated between history and science. However, he was strongly interested in natural history and that decided him to take up biology and chemistry. After he had become a scholar at Trinity, his future zoology teacher, Carl Pantin, advised him to learn as much mathematics and physics as he could. This was good, if painful, advice which has kept him busy ever since. ... Hodgkin spent several weeks with K. S. Cole at Woods Hole, and there he learnt how to dissect squid axons.

R. Fitzhugh J. Nagumo, et al.

The Fitzhugh-Nagumo Equations

$$y' = \frac{1}{30}\,(7 - 10\,v - 8\,y)$$

$$v' = 3\,y - v\,(v^2 - 3) + \sigma$$

↑

stimulus

Spruce Budworms in Canadian Forests

[*Nature*, Vol. 269, p. 471 (1977)]

y = foliage density, w = budworm density

$$\frac{dy}{dt} = k_1 y(1-y) - \mu w$$

$$\frac{dw}{dt} = k_2 w\left(1 - \frac{w}{\alpha y}\right) - \rho \frac{w^2}{\beta^2 y^2 + w^2}$$

↑

predation by birds